

Structural Arithmetic Substrate (SA)

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Definition (SA)

The Structural Arithmetic Substrate (SA) is a realized, discrete structural object defined as follows.

1. Carrier

SA is defined on a finite or extensible grid of integer coordinate points.

The grid may be two-dimensional or higher-dimensional.

2. Occupancy

Each coordinate point in the grid is assigned a binary state:

occupied (present), or unoccupied
(absent).

This assignment defines the realized structure of SA.

3. Generation rule

Occupancy is generated by a local relational rule acting on nearby points.

The rule:

- does not test for primality,
- does not use symbolic arithmetic,
- does not rely on analytic continuation,
- does not assume global classification,
- does not invoke oracles or choice principles.

The structure is produced entirely by construction.

4. Realizability constraint

SA consists only of the constructed structure itself.

No propositions, equations, or truth claims about the structure are part of SA.

5. Typing (FM)

SA is a Layer-3 object under Falsifiability Mechanics:

- it is structurally true,
unfalsifiable,
prior to propositional claims.

Notes (formal)

SA is not invariant-complete.

SA admits refinement.

Arithmetic properties are emergent descriptions of SA, not primitives of SA.

Structural Invariant Arithmetic Substrate (SIAS)

Definition (SIAS)

The Structural Invariant Arithmetic Substrate (SIAS) is the invariant-complete refinement of an SA instance.

1. Base object

SIAS is defined on an existing SA structure.

2. Admissible transformations

A transformation is admissible if it preserves realized structure and does not discard information.

Examples

include:

- scaling

- ,

- rotation,

- axis compression or expansion,

- dimensional lifting,

- symmetry completion.

Symbolic argument **alone** is not sufficient; transformations must be realizable.

3. Invariant criterion

A structural feature is an SIAS invariant if and only if:

The feature remains structurally identical (up to relabeling) under every **admissible** transformation.

4. Completion

SIAS consists of:

the underlying SA structure, and
all and only those features that satisfy the invariant criterion.

5. Verification rule

Invariance must be **verified by** explicit realization under transformation.

Symbolic **proof alone** is insufficient.

6. Typing (FM)

SIAS is a Layer-3 invariant-certified object, suitable for:

diagnosing boundary-typed claims,
diagnosing ill-typed global closures,

analyzing admissibility of universal statements.

Notes (formal)

SIAS is not a proof system.

SIAS does not assert global closure results.

Arithmetic statements are descriptive summaries of SIAS invariants.

Relationship Between SA and SIAS

- SA refines into SIAS; the relationship is not symmetric.
Every SIAS invariant restricts to a feature of SA.
Not every feature of SA survives as an SIAS invariant.
- Distinguishing SA from SIAS is required for correct FM typing.

Minimal Interpretation Summary

SA answers: What structure is realized?

SIAS answers: Which features of that structure survive all realization-preserving transformations?

- Neither asserts truth of conjectures.
- Both operate prior to symbolic arithmetic claims.